Caches in WCET Analysis

Predictability, Competitiveness, Sensitivity

Jan Reineke

November 7^{th}, 2008

Outline

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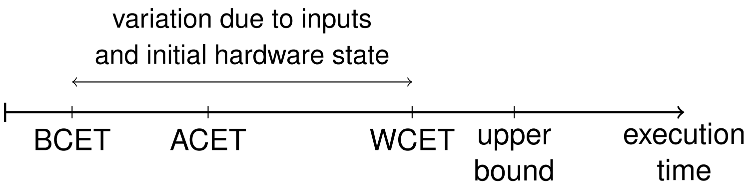
1. Introduction
   - WCET Analysis
   - Caches and Cache Analysis
2. Predictability Metrics
3. Relative Competitiveness
4. Sensitivity – Caches and Measurement-Based Timing Analysis
5. Summary

WCET Analysis

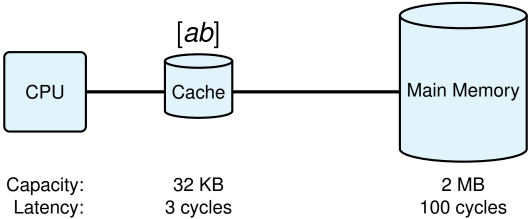
- Controllers in planes, cars, plants, ... often have to satisfy hard real-time constraints

⟶ Need to statically derive upper bounds on WCETs of tasks



Caches

- How they work:
  - dynamically and transparently
  - managed by replacement policy



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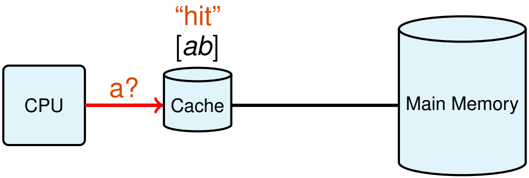

Capacity:	32 KB	2 MB
Latency:	3 cycles	100 cycles

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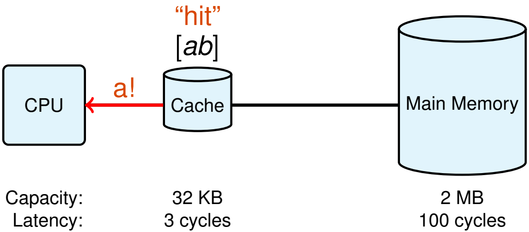

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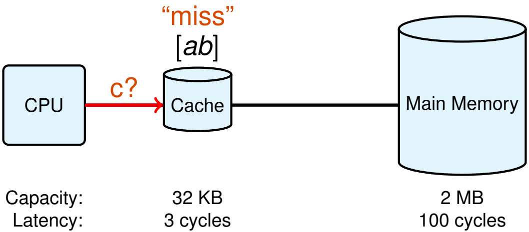

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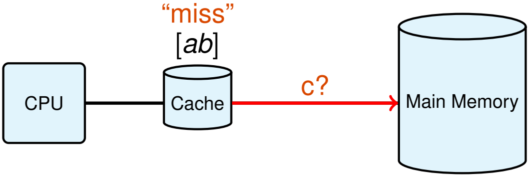

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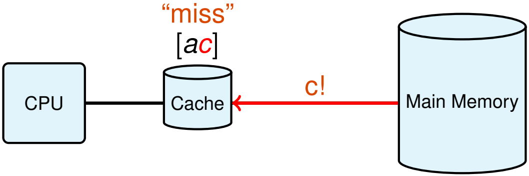

	Cache	Main Memory
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Latency:	3 cycles	100 cycles

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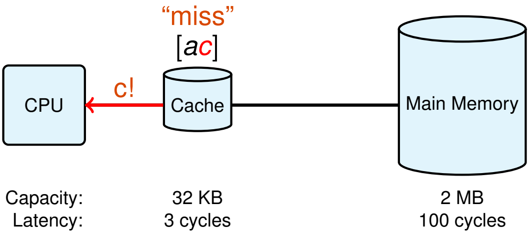


⟶ Cache analysis statically derives guarantees on cache behavior

Cache Analysis

Two types of cache analyses:

1. Local guarantees: classification of individual accesses
   - May-Analysis $\longrightarrow$ Overapproximates cache contents
   - Must-Analysis $\longrightarrow$ Underapproximates cache contents
2. Global guarantees: bounds on cache hits/misses

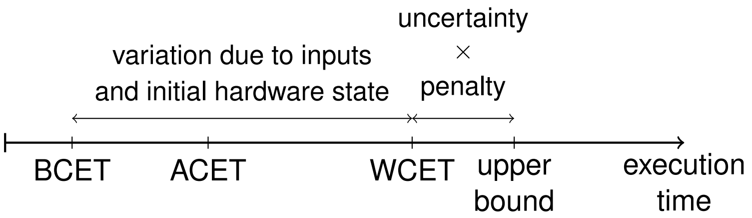
Cache Replacement Policies

- Least Recently Used (LRU) used in
  INTEL PENTIUM I and MIPS 24K/34K
- First-In First-Out (FIFO or Round-Robin) used in
  MOTOROLA POWERPC 56X, INTEL XSCALE, ARM9, ARM11
- Pseudo-LRU (PLRU) used in
  INTEL PENTIUM II-IV and POWERPC 75X
- Most Recently Used (MRU) as described in literature

- Cache analyses almost exclusively for LRU
- In practice: FIFO, PLRU, Pseudo Round-Robin, ...

Uncertainty in WCET Analysis

- Precision of WCET analysis determined by amount of uncertainty
- Uncertainty in cache analysis depends on replacement policy



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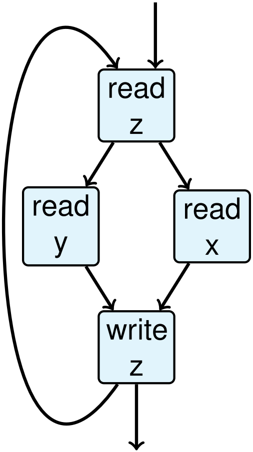

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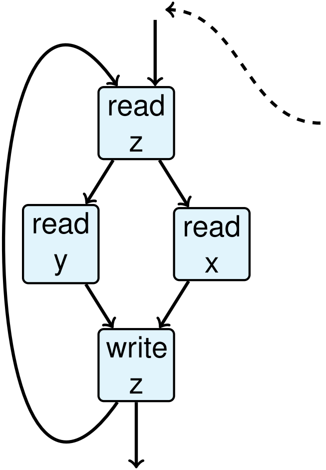

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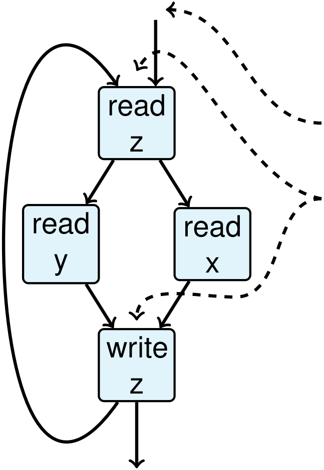
Uncertainty in Cache Analysis



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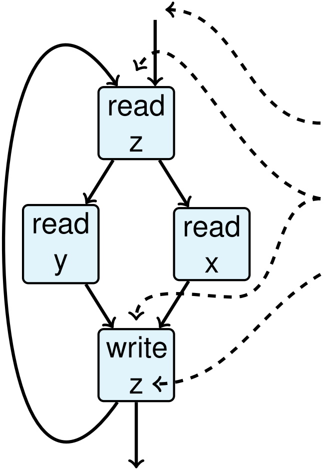

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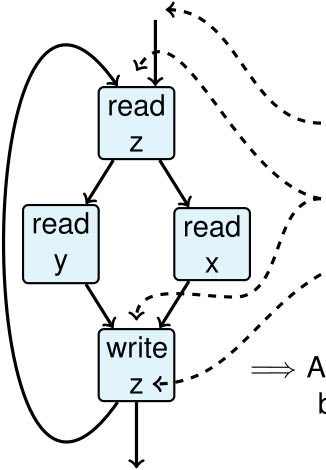

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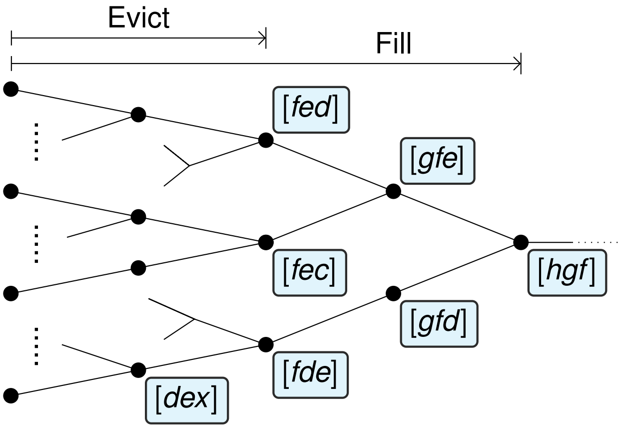
$\Longrightarrow$ Amount of uncertainty determined by ability to recover information

Predictability Metrics



Sequence: $\langle$a, . . . , e, f, g, h$\rangle$

Meaning of Metrics

- Evict
  - Number of accesses to obtain *any may*-information.
  - I.e. when can an analysis predict any cache misses?
- Fill
  - Number of accesses to complete *may*- and *must*-information.
  - I.e. when can an analysis predict each access?

$\longrightarrow$ Evict and Fill bound the precision of *any* static cache analysis.
Can thus serve as a benchmark for analyses.

Evaluation of Policies

Policy	Evict(k)	Fill(k)	Evict(8)	Fill(8)
LRU	k	k	8	8
FIFO	$2k-1$	$3k-1$	15	23
MRU	$2k-2$	$\infty/3k-4$	14	$\infty/20$
PLRU	$\frac{k}{2}\log_2 k + 1$	$\frac{k}{2}\log_2 k + k - 1$	13	19

- LRU is optimal w.r.t. metrics.
- Other policies are much less predictable.

$\longrightarrow$ Use LRU.

- How to obtain *may*- and *must*-information within the given limits for other policies?

Outline

Relative Competitiveness

- Competitiveness (Sleator and Tarjan, 1985):
  worst-case performance of an online policy *relative to the optimal offline policy*
  - used to evaluate online policies

- Relative competitiveness (Reineke and Grund, 2008):
  worst-case performance of an online policy *relative to another online policy*
  - used to derive local and global cache analyses

Definition – Relative Miss-Competitiveness

Notation

$m_{\mathbf{P}}(p, s) \quad = \quad$ *number of misses that policy* **P** *incurs on access sequence* $s \in M^*$ *starting in state* $p \in C^{\mathbf{P}}$

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Definition (Relative miss competitiveness)

Policy **P** is (k, c)-miss-competitive relative to policy **Q** if

$$m_{\mathbf{P}}(p, s) \leq k \cdot m_{\mathbf{Q}}(q, s) + c$$

for all access sequences $s \in M^*$ and cache-set states $p \in C^{\mathbf{P}}, q \in C^{\mathbf{Q}}$ that are compatible $p \sim q$.

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Definition (Competitive miss ratio of **P** relative to **Q**)

The smallest k, s.t. **P** is (k, c)-miss-competitive rel. to **Q** for some c.

Example – Relative Miss-Competitiveness

P is $(3, 4)$-miss-competitive relative to **Q**.
If **Q** incurs x misses, then **P** incurs at most $3 \cdot x + 4$ misses.

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Worst: **P** is $(0, 0)$-hit-competitive relative to **Q**.
Analogue to ∞-miss-competitiveness.

Local Guarantees: $(1, 0)$-Competitiveness

Let **P** be $(1, 0)$-competitive relative to **Q**:

$$m_{\mathbf{P}}(p, s) \leq 1 \cdot m_{\mathbf{Q}}(q, s) + 0$$
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1. If **Q** “hits”, so does **P**, and
2. if **P** “misses”, so does **Q**.

As a consequence,

1. a *must*-analysis for **Q** is also a *must*-analysis for **P**, and
2. a *may*-analysis for **P** is also a *may*-analysis for **Q**.

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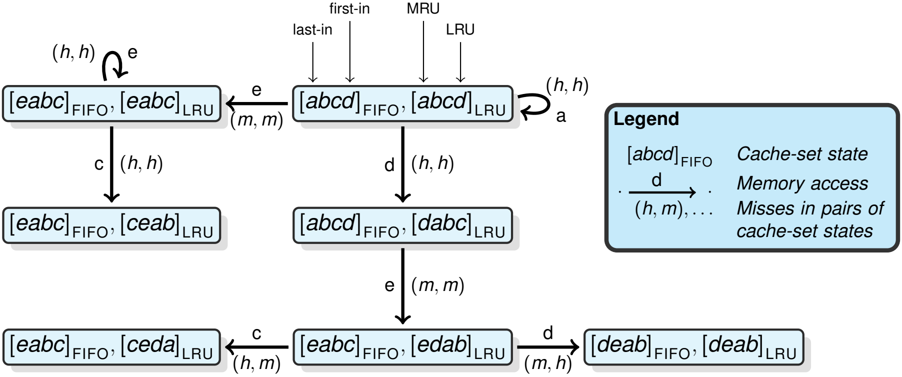
3. Calculate global guarantee on the number of misses for **P** using the global guarantee for **Q** and the competitiveness results of **P** relative to **Q**.

$$\mathbf{m_P \leq k \cdot m_Q + c} \;\; \mathbf{m_Q(T)} = \mathbf{m_P(T)}$$

Relative Competitiveness – Automatic Computation

P and **Q** (here: FIFO and LRU) induce transition system:

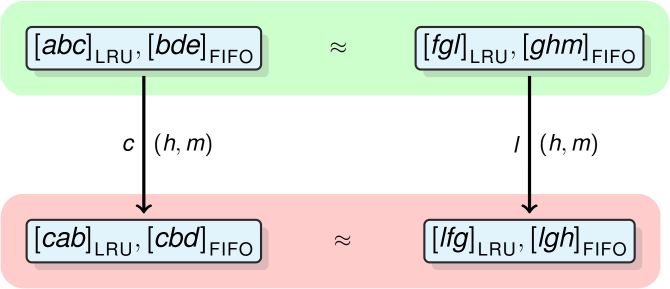


Competitive miss ratio = maximum ratio of misses in policy **P** to misses in policy **Q** in transition system

Transition System is ∞ Large

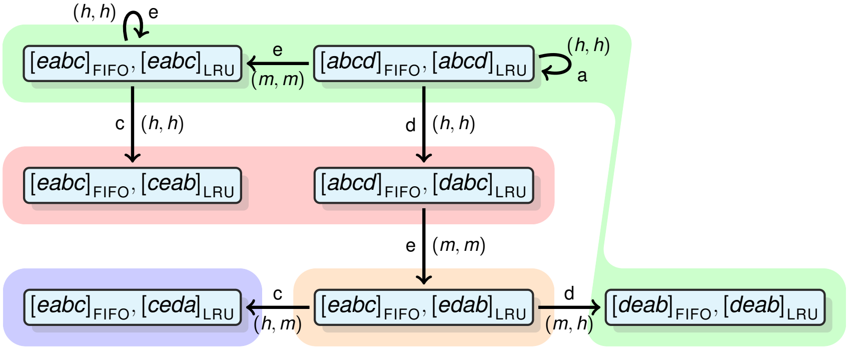
Problem: The induced transition system is ∞ large.

Observation: Only the *relative positions* of elements matter:



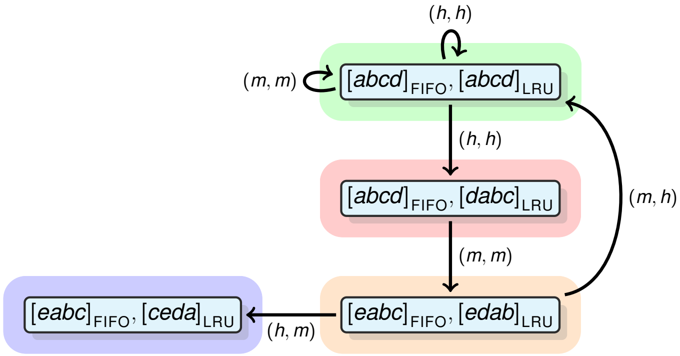
Solution: Construct *finite* quotient transition system.

≈-Equivalent States in Running Example



Finite Quotient Transition System

Merging $\approx$-equivalent states yields a finite quotient transition system:



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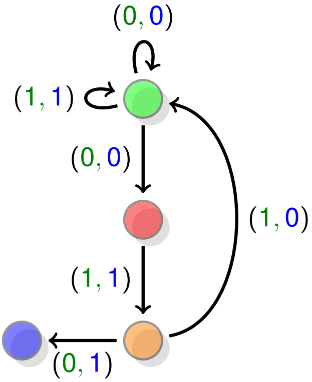

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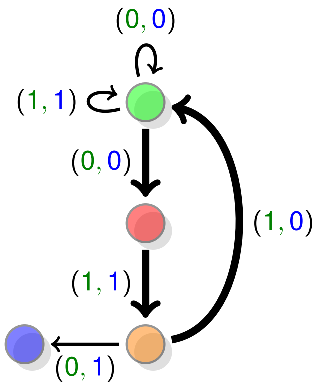

Maximum cycle ratio = $\frac{0+1+1}{0+1+0} = 2$

Tool Implementation

- Implemented in Java
- Interface for replacement policies

- Fully automatic
- Provides example sequences for competitive ratio and constant

- Analysis usually practically feasible up to associativity 8
  - limited by memory consumption
  - depends on similarity of replacement policies

Generalizations

Identified patterns and proved generalizations by hand.

Aided by example sequences generated by tool.

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$$\text{LRU}(2k-1) \text{ is } (1,0) \text{ comp. rel. to } \text{FIFO}(k), \text{ and}$$

$$\text{LRU}(2k-2) \text{ is } (1,0) \text{ comp. rel. to } \text{MRU}(k).$$

$\longrightarrow$ LRU-*may*-analysis can be used for FIFO and MRU

$\longrightarrow$ optimal with respect to predictability metrics

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FIFO-*may*-analysis used in the analysis of the branch target buffer of the MOTOROLA POWERPC 56X.

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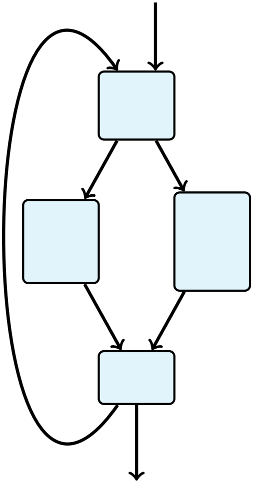
Measurement-Based Timing Analysis

- Run program on a number of inputs and initial states.
- Combine measurements for basic blocks to obtain WCET estimation.
- Sensitivity Analysis demonstrates this approach may be dramatically wrong.



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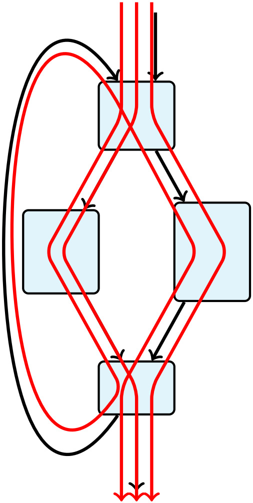
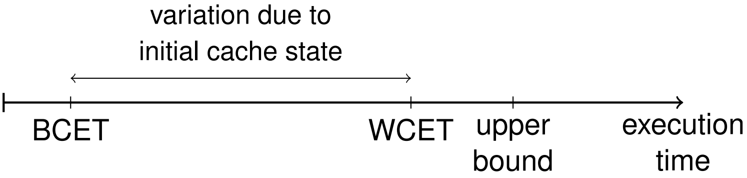


Influence of Initial Cache State



Definition (Miss sensitivity)

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for all access sequences $s \in M^*$ and cache-set states $p, p' \in C^{\mathbf{P}}$.

Sensitivity Results

Policy	2	3	4	5	6	7	8
LRU	$1, 2$	$1, 3$	$1, 4$	$1, 5$	$1, 6$	$1, 7$	$1, 8$
FIFO	$2, 2$	$3, 3$	$4, 4$	$5, 5$	$6, 6$	$7, 7$	$8, 8$
PLRU	$1, 2$	$-$	∞	$-$	$-$	$-$	∞
MRU	$1, 2$	$3, 4$	$5, 6$	$7, 8$	MEM	MEM	MEM

- LRU is optimal. Performance varies in the least possible way.
- For FIFO, PLRU, and MRU the number of misses may vary strongly.
- Case study based on simple model of execution time by Hennessy and Patterson (2003):
  WCET may be 3 times higher than a measured execution time for 4-way FIFO.

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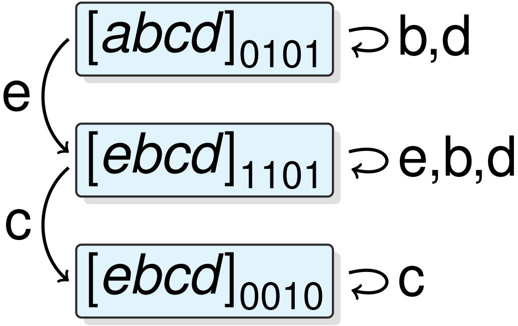
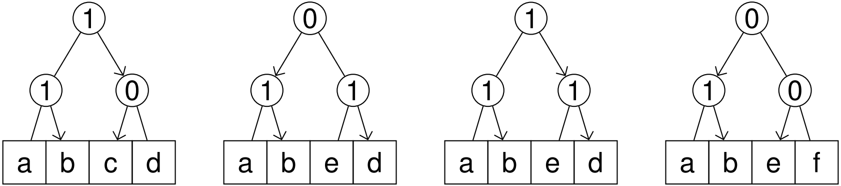
Most-Recently-Used – MRU

MRU-bits record whether line was recently used



⟶ Never converges

Pseudo-LRU – PLRU



Initial cache-set state $[a, b, c, d]_{110}$.

After a miss on *e*. State: $[a, b, e, d]_{011}$.

After a hit on *a*. State: $[a, b, e, d]_{111}$.

After a miss on *f*. State: $[a, b, e, f]_{010}$.

Hit on *a* “rejuvenates” neighborhood; “saves” *b* from eviction.

May- and Must-Information

$$May^{\mathbf{P}}(s) := \bigcup_{p \in C^{\mathbf{P}}} CC_{\mathbf{P}}(update_{\mathbf{P}}(p, s))$$

$$Must^{\mathbf{P}}(s) := \bigcap_{p \in C^{\mathbf{P}}} CC_{\mathbf{P}}(update_{\mathbf{P}}(p, s))$$

$$may^{\mathbf{P}}(n) := \left|May^{\mathbf{P}}(s)\right|, \text{where } s \in S^{\neq} \subsetneq M^*, |s| = n$$

$$must^{\mathbf{P}}(n) := \left|Must^{\mathbf{P}}(s)\right|, \text{where } s \in S^{\neq} \subsetneq M^*, |s| = n$$

$S^{\neq}$: set of finite access sequences with pairwise different accesses

Definitions of Metrics

$$\text{Evict}^{\mathbf{P}} := \min\left\{n \mid may^{\mathbf{P}}(n) \leq n\right\},$$

$$\text{Fill}^{\mathbf{P}} := \min\left\{n \mid must^{\mathbf{P}}(n) = k\right\},$$

where k is **P**'s associativity.

Relation: Pred. Metrics $\leftrightarrow$ Rel. Competitiveness

Let $P(k)$ be $(1,0)$-miss-competitive relative to policy $Q(l)$, then

(i) $Evict^P(k) \geq Evict^Q(l)$,

(ii) $mls^P(k) \geq mls^Q(l)$.

Alternative Pred. Metrics $\leftrightarrow$ Rel. Competitiveness

Let l be the smallest associativity, such that $\mathrm{LRU}(l)$ is $(1,0)$-miss-competitive relative to $P(k)$. Then

$$\text{Alt-Evict}^{P}(k) = l.$$

Let l be the greatest associativity, such that $P(k)$ is $(1,0)$-miss-competitive relative to $\mathrm{LRU}(l)$. Then

$$\text{Alt-mls}^{P}(k) = l.$$

Size of Transition System

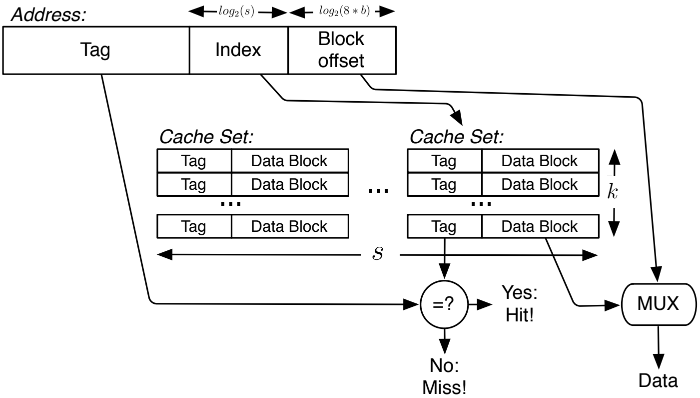
$$\underbrace{2^{l+l'}}_{\substack{\text{status bits}\\ \text{of } \mathbf{P} \text{ and } \mathbf{Q}}} \cdot \underbrace{\sum_{i=0}^{k} \binom{k}{i}}_{\text{non-empty lines in } \mathbf{P}} \cdot \underbrace{\sum_{i'=0}^{k'} \binom{k'}{i'}}_{\text{non-empty lines in } \mathbf{Q}} \cdot \underbrace{\sum_{j=0}^{\min\{i,i'\}} \binom{i}{j}\binom{i'}{j} j!}_{\substack{\text{number of overlappings}\\ \text{in non-empty lines}}}$$

$$\begin{aligned}\sum_{j=0}^{\min\{k,k'\}} \binom{k}{j}\binom{k'}{j} j! &\leq k! \cdot k'! \sum_{j=0}^{\min\{k,k'\}} \frac{1}{(k-j)!j!(k'-j)!} \\ &\leq k! \cdot k'! \sum_{j=0}^{\infty} \frac{1}{j!} = e \cdot k! \cdot k'!\end{aligned}$$

This can be bounded by

$$2^{l+l'+k+k'} \leq |(C_k^l \times C_{k'}^{l'})/\approx| \leq 2^{l+l'+k+k'} \cdot \underbrace{e \cdot k! \cdot k'!}_{\text{bound on number of overlappings}}$$

Set-Associative Caches



Compatible States

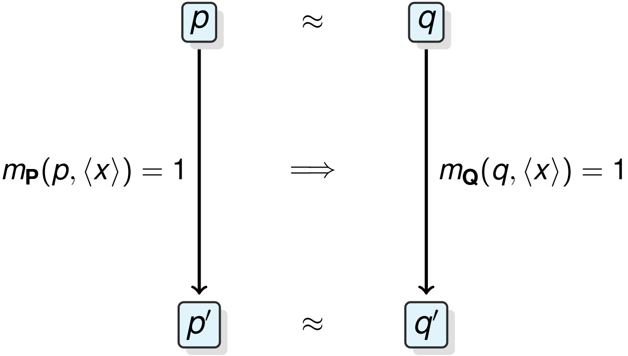
$$i^{\mathbf{P}} = [\bot\bot\bot\bot]_{\mathbf{P}} \approx i^{\mathbf{Q}} = [\bot\bot\bot\bot]_{\mathbf{Q}}$$

$$\mathit{update}_{\mathbf{P}}(i^{\mathbf{P}}, s) \qquad \mathit{update}_{\mathbf{Q}}(i^{\mathbf{Q}}, s)$$

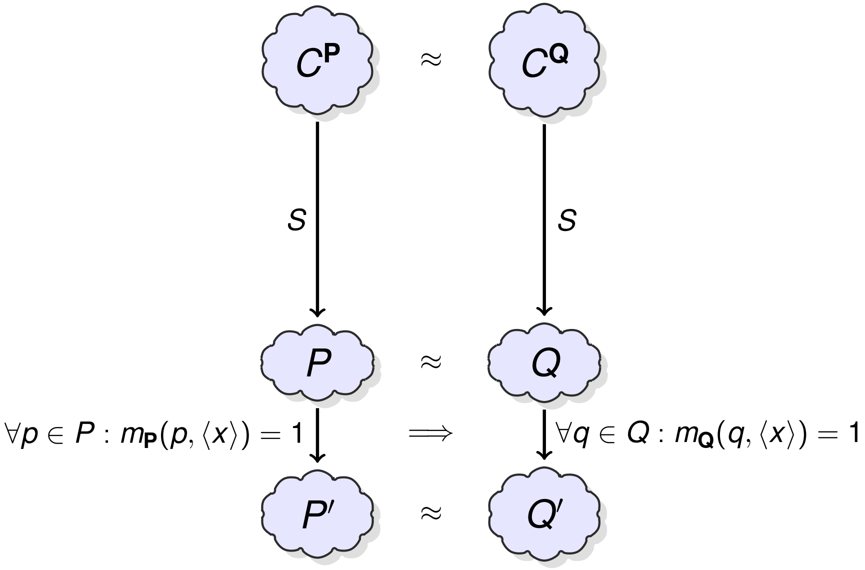
$$p \approx q$$

$(1,0)$-Competitiveness and May/Must-Analyses

Let **P** be $(1,0)$-competitive relative to **Q**, then



$(1,0)$-Competitiveness and May/Must-Analyses



Case Study: Impact of Sensitivity

- Simple model of execution time from Hennessy & Patterson (2003)
- CPI_{hit} = Cycles per instruction assuming cache hits only
- $\frac{\text{Memory accesses}}{\text{Instruction}}$ including instruction and data fetches

$$\frac{T_{wc}}{T_{meas}} = \frac{\text{CPI}_{hit} + \frac{\text{Memory accesses}}{\text{Instruction}} \times \text{Miss rate}_{wc} \times \text{Miss penalty}}{\text{CPI}_{hit} + \frac{\text{Memory accesses}}{\text{Instruction}} \times \text{Miss rate}_{meas} \times \text{Miss penalty}}$$

$$= \frac{1.5 + 1.2 \times 0.20 \times 50}{1.5 + 1.2 \times 0.05 \times 50} = \frac{13.5}{4.5} = 3$$