Workshop “Trustworthy Software” 2006

Shape Analysis of Sets

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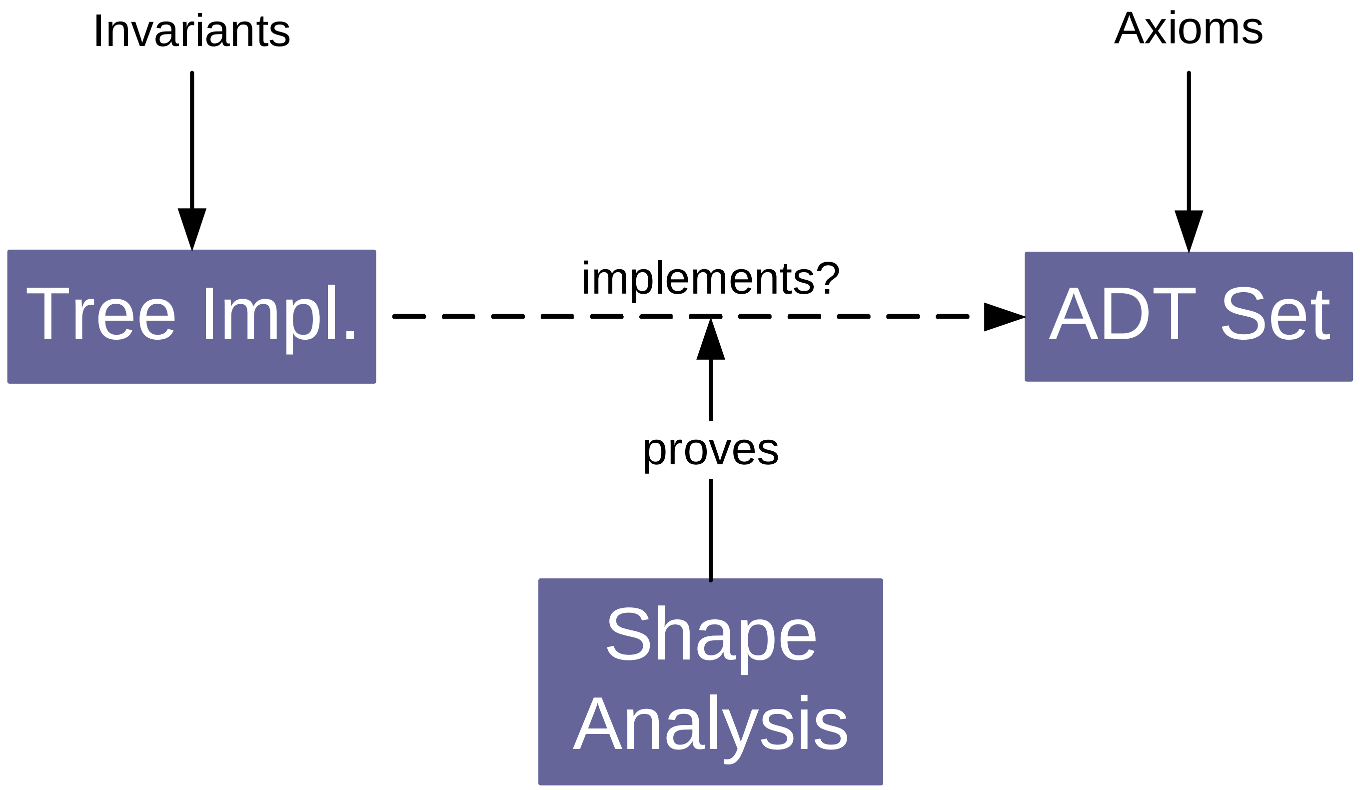
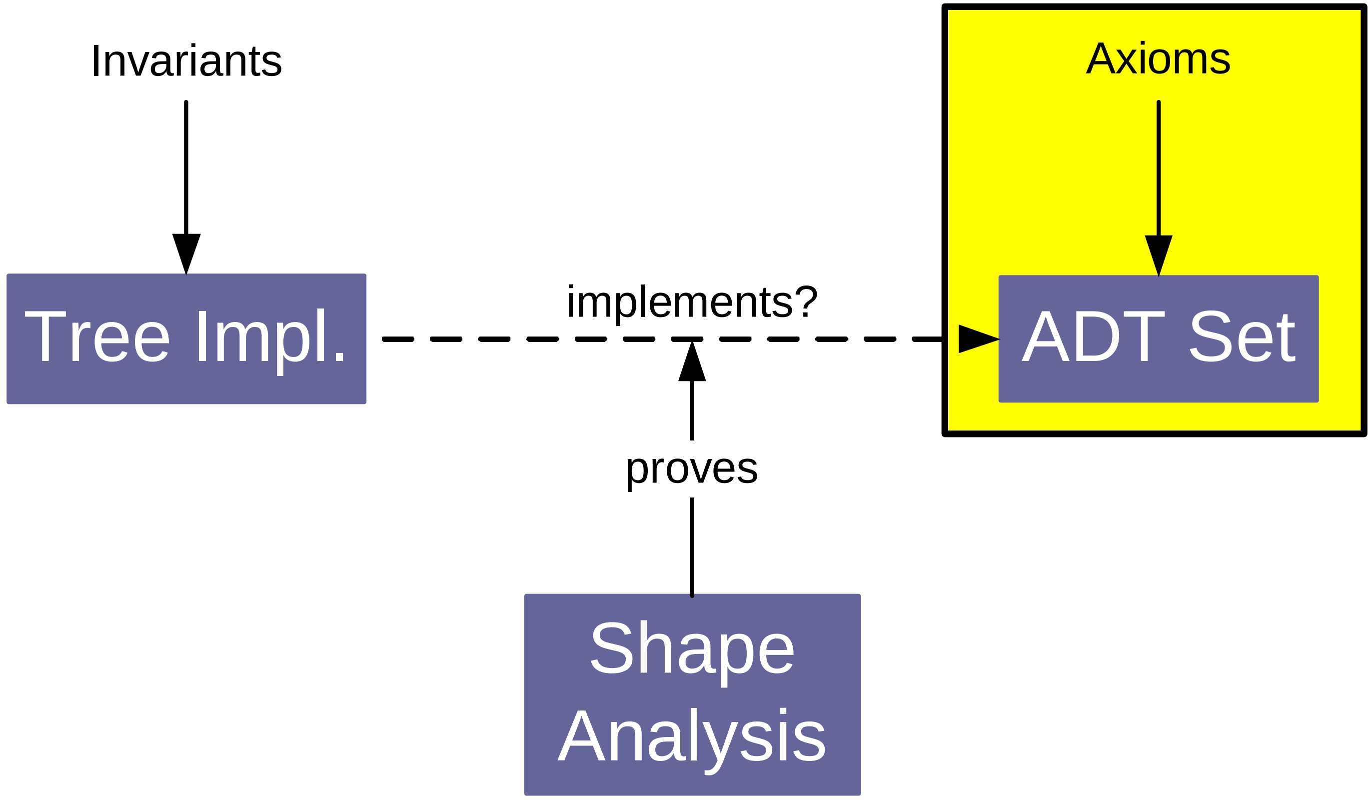
The Big Picture



ADT Set


Invariants
Tree Impl.
implements?
Axioms
ADT Set
proves
Shape
Analysis

ADT Set - Algebraic Specification

- Collection of elements of a certain type
- Operations:

$$
\begin{array}{lcl}
\cdot\texttt{.insert}(\cdot) & : & \text{set} \times \text{element} \rightarrow \text{set} \\
\cdot\texttt{.remove}(\cdot) & : & \text{set} \times \text{element} \rightarrow \text{set} \\
& \ldots &
\end{array}
$$

- Predicates:

$$
\begin{array}{lcl}
. \in . & : & \text{element} \times \text{set} \\
. \subseteq . & : & \text{set} \times \text{set} \\
. = . & : & \text{set} \times \text{set}
\end{array}
$$

ADT Set Axioms (selection)

$$a \in s.\mathtt{insert}(b) \leftrightarrow a =_{el} b \vee a \in s, \quad (1)$$

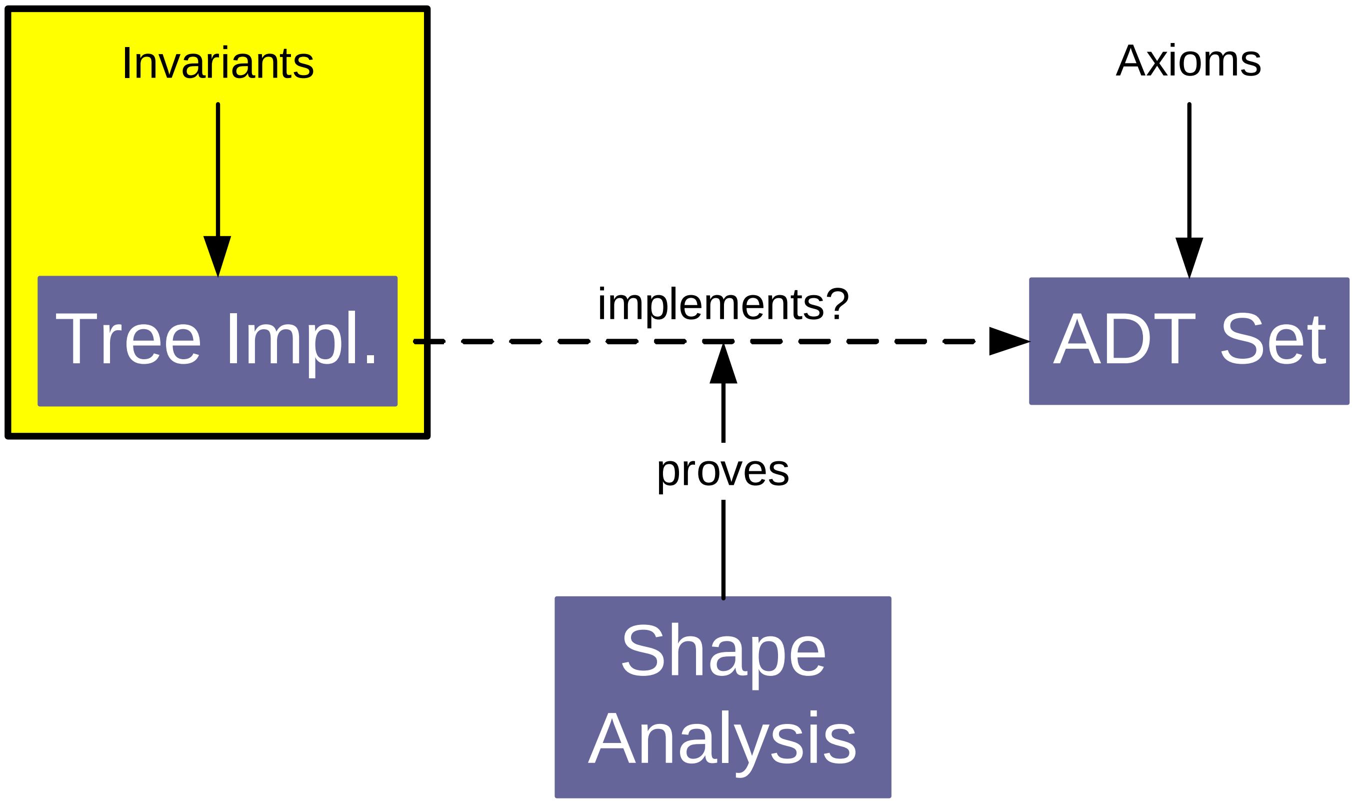
$$a \in s.\mathtt{remove}(b) \leftrightarrow a \neq_{el} b \wedge a \in s, \quad (2) \quad \hookleftarrow$$

$$s \subseteq s' \leftrightarrow (\forall a. a \in s \rightarrow a \in s'), \quad (3)$$

$$s = s' \leftrightarrow (s \subseteq s' \wedge s' \subseteq s), \quad (4)$$

$\ldots$

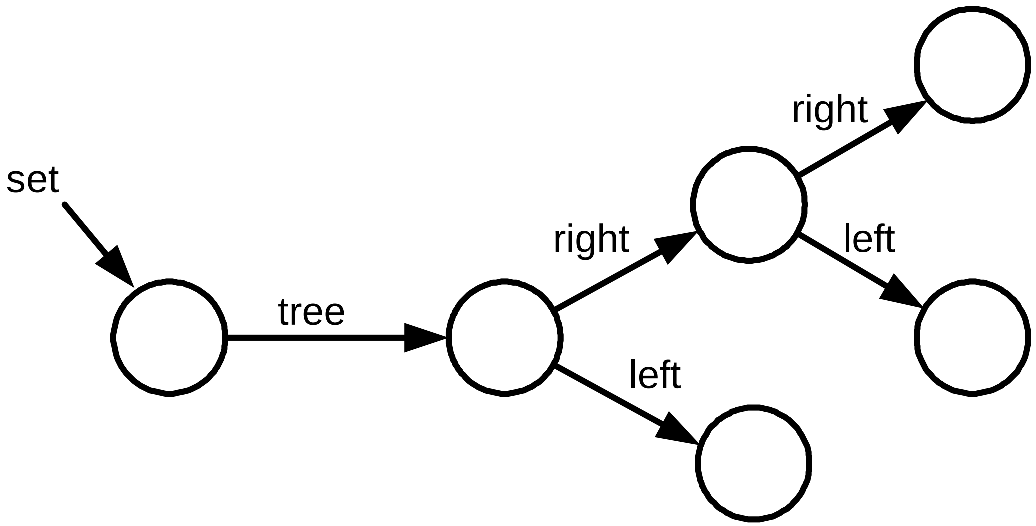
Tree-based Implementation



Structure Declarations

```
typedef struct Tree
{
  void* data;
  struct Tree* left;
  struct Tree* right;
} List;
```

```
typedef struct Set
{
  List* tree;
  int (*compare)(void*, void*);
  int size;
} Set;
```

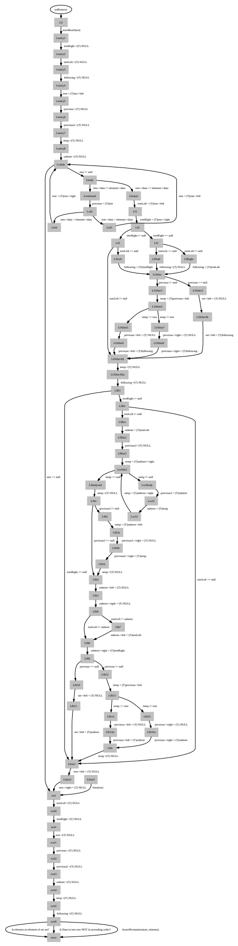


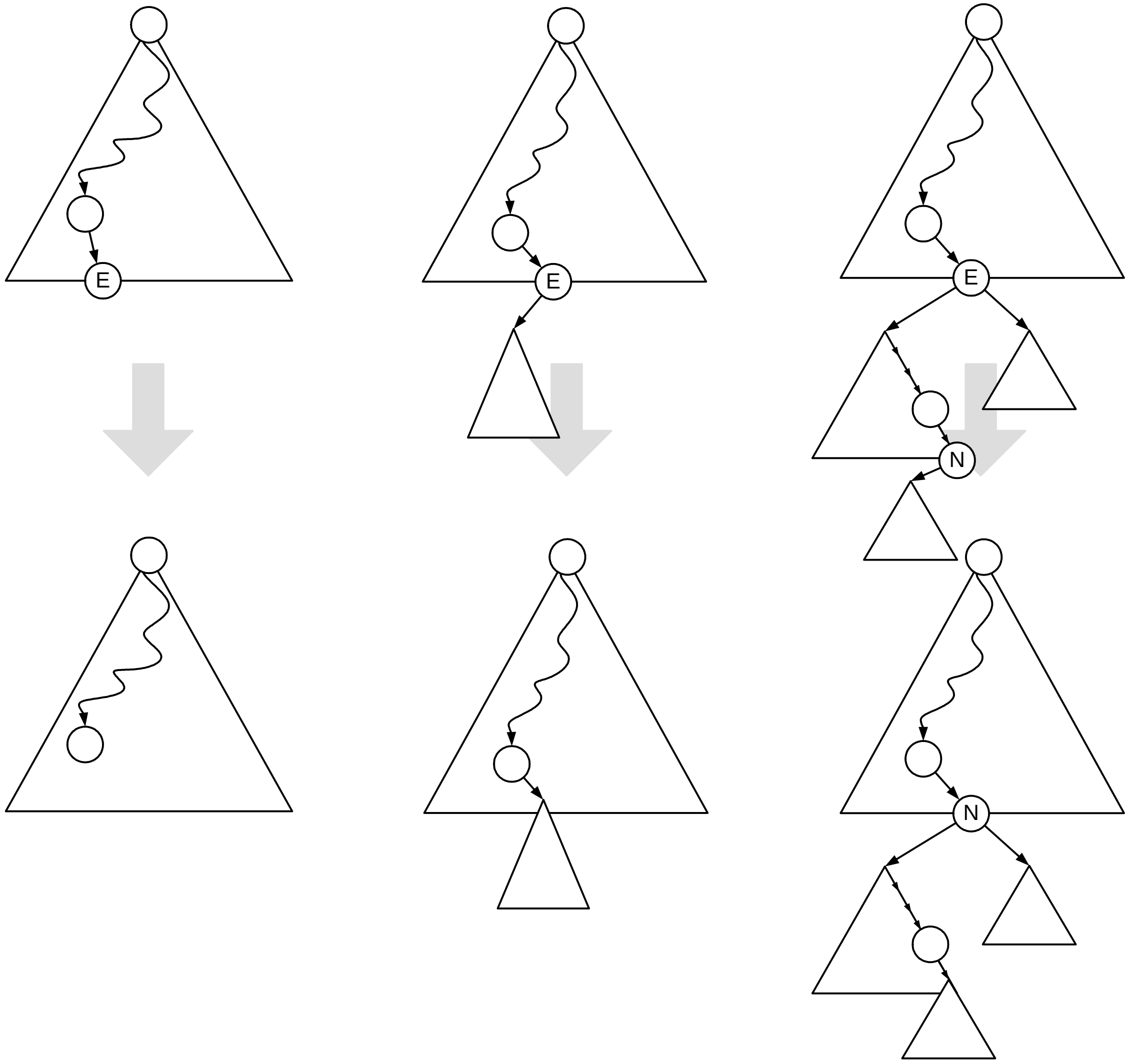
Data Structure Invariants

- The tree is in fact a tree ;-)
  i.e. every node is reachable from set and pointed to from exactly one other node
- The tree is ordered:
  left descendants are smaller, right descendants larger

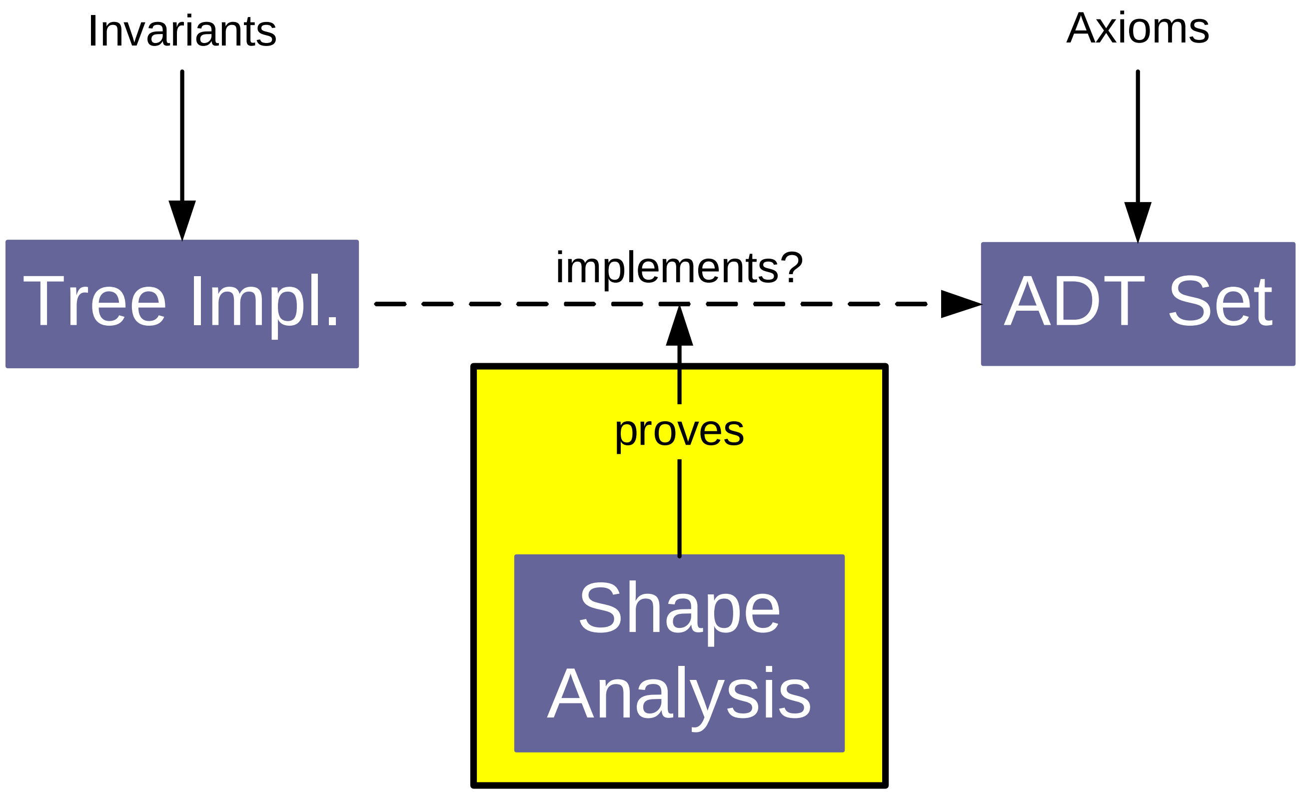
$\Rightarrow$ both are formalized by instrumentation predicates

Removing an Element





Shape Analysis



Proving Compliance to ADT Axioms
Problem 1

- Problem 1: Axioms relate different routines of the set implementation: $\in$ and `remove`
  $a \in s$ .remove $(b) \leftrightarrow a \neq_{el} b \wedge a \in s$
- Solution:
  - Represent $\in$ predicate by $isElement$ instrumentation predicate
  - Prove the equivalence of $\in$-implementation and $isElement$
  - Prove compliance of `remove`-implementation to axiom in terms of $isElement$

Proving Compliance to ADT Axioms
Problem 2

- Problem 2: Axioms relate state of predicates before and after execution:
  $a \in s.\text{remove}(b) \leftrightarrow a \neq_{el} b \wedge a \in s$
- Solution: Remember old element relation $isElementOld$
  - Predicate is fixed before invocation of method
  - Allows to compare new and old values of element property
  - Primed vs. Unprimed versions

So what do our analyses really prove?

Axiom: $a \in s.\texttt{remove}(b) \leftrightarrow a \neq_{el} b \wedge a \in s$

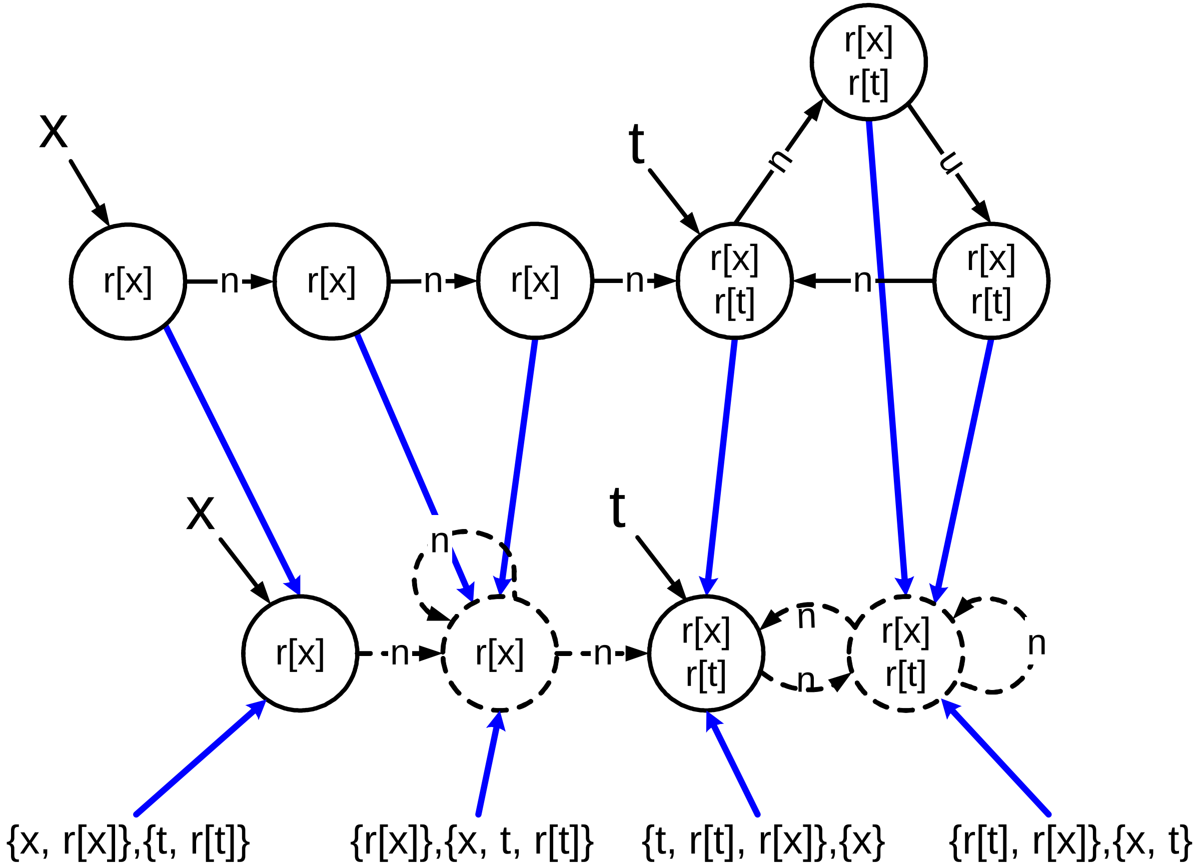
1. $isElement(a, s) \Leftrightarrow a \in s$
2. After executing $s.\texttt{remove}(b)$ we check $isElement(a, s) \leftrightarrow a \neq_{el} b \wedge isElementOld(a, s)$

Recent Developments in Shape Analysis

- very precise Shape Analysis algorithms
  $\longrightarrow$ able to prove partial correctness of programs: bubble-sort, insertion-sort, etc. (LARSW00)
- instantiations of a Parametric Shape Analysis Framework of (SRW02) that use logical structures to represent states
- has been implemented in a tool called TVLA (= Three-Valued-Logic Analyzer)

Canonical Abstraction

- Collapse individuals that agree on unary predicates.



- At most $3^{|U|}$ abstract individuals.

How to make analyses precise
Key Predicates

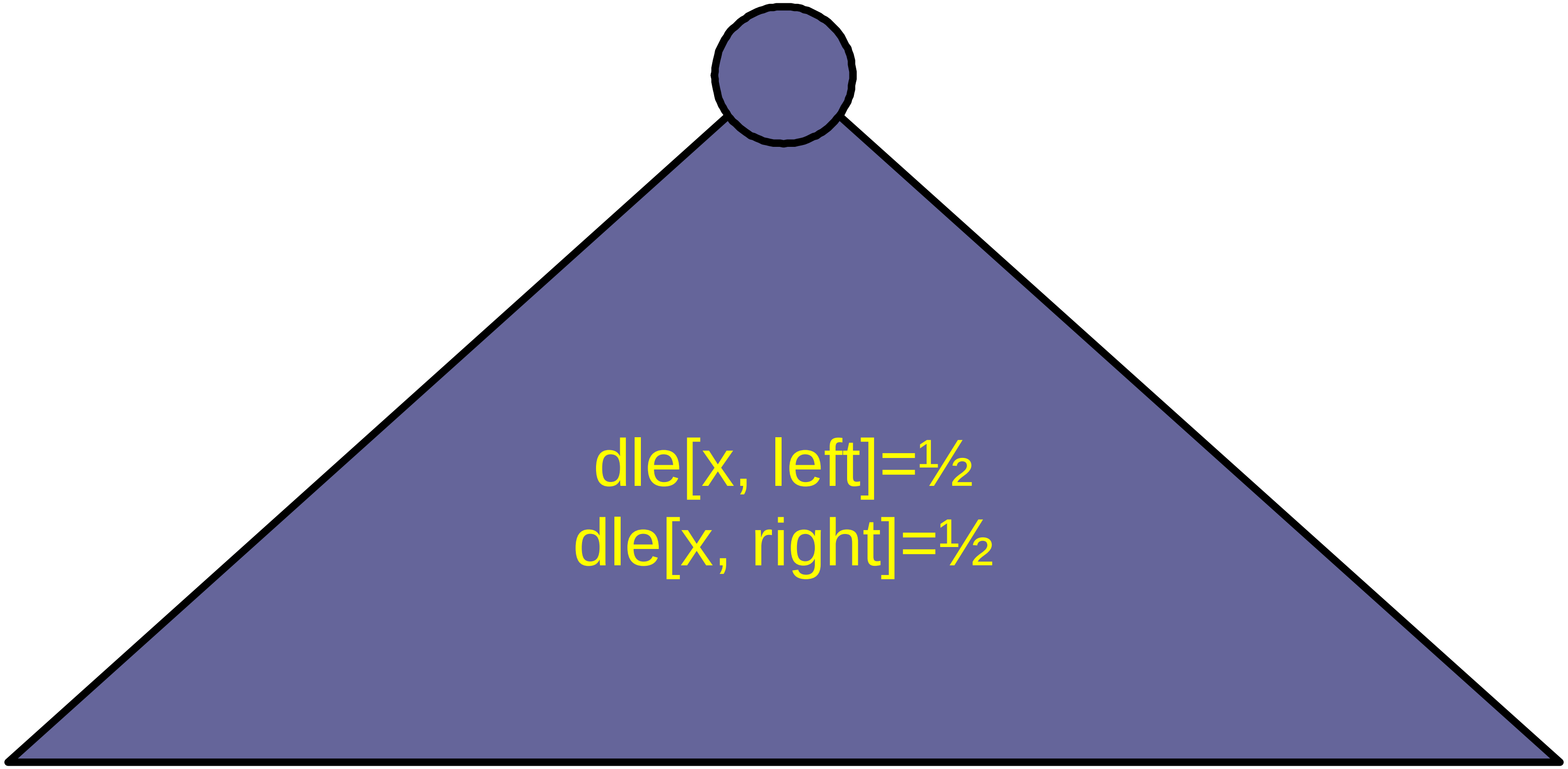
- Model $data$-field indirectly
  dle-predicate - "data less or equal"
  stores value of $compare$-function

- Capture dle-relation with nodes pointed to by variables: $dle[variable, left]$- and $dle[variable, right]$-predicate family

- Keep precise reachability information through:
  $downStar[left]$, $downStar[right]$

Predicates - dle(var,left) / dle(var, right)

$$dle[x, left](v) = \exists v_1.(x(v1) \wedge dle(v, v1) \wedge \neg dle(v1, v))$$
$$dle[x, right](v) = \exists v_1.(x(v1) \wedge \neg dle(v, v1) \wedge dle(v1, v))$$



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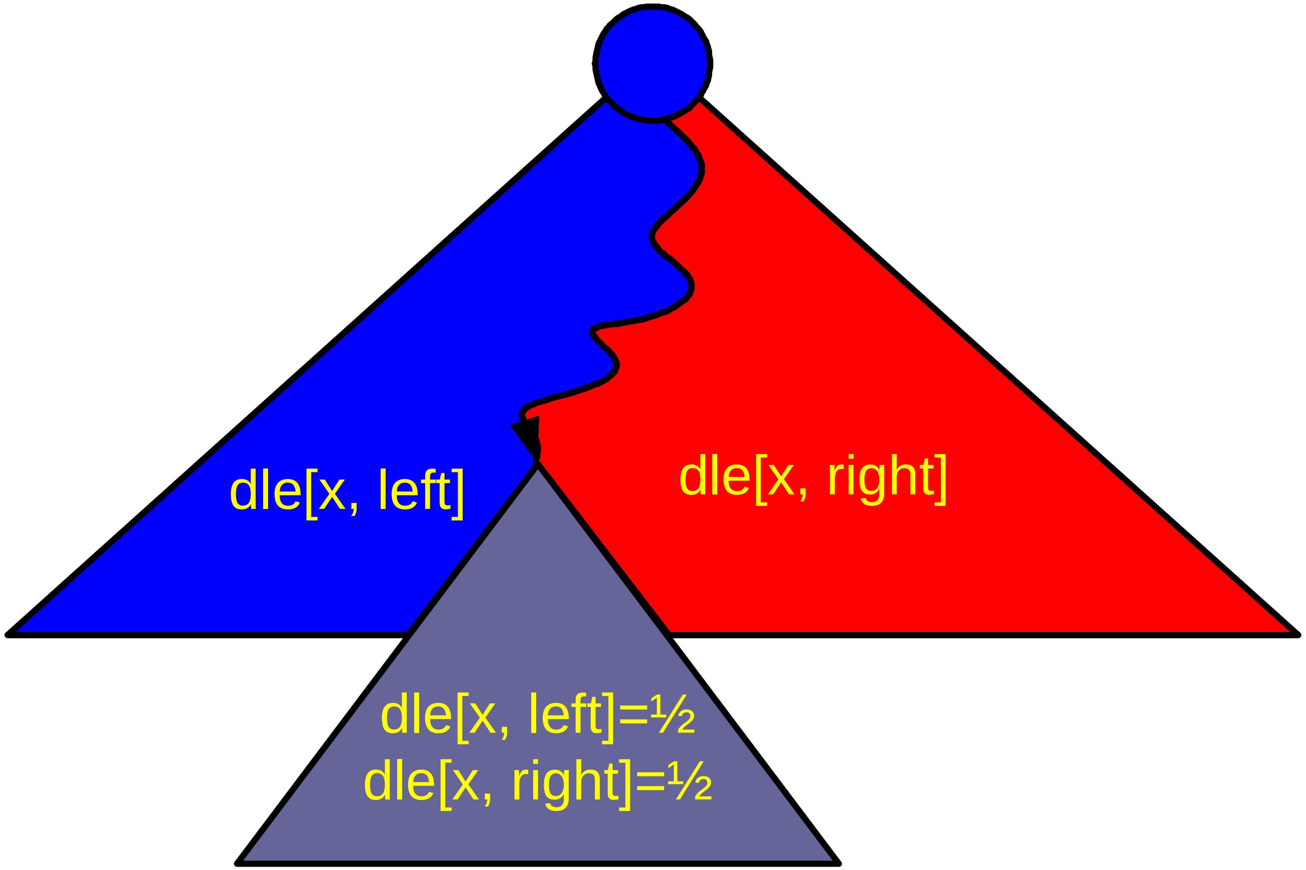

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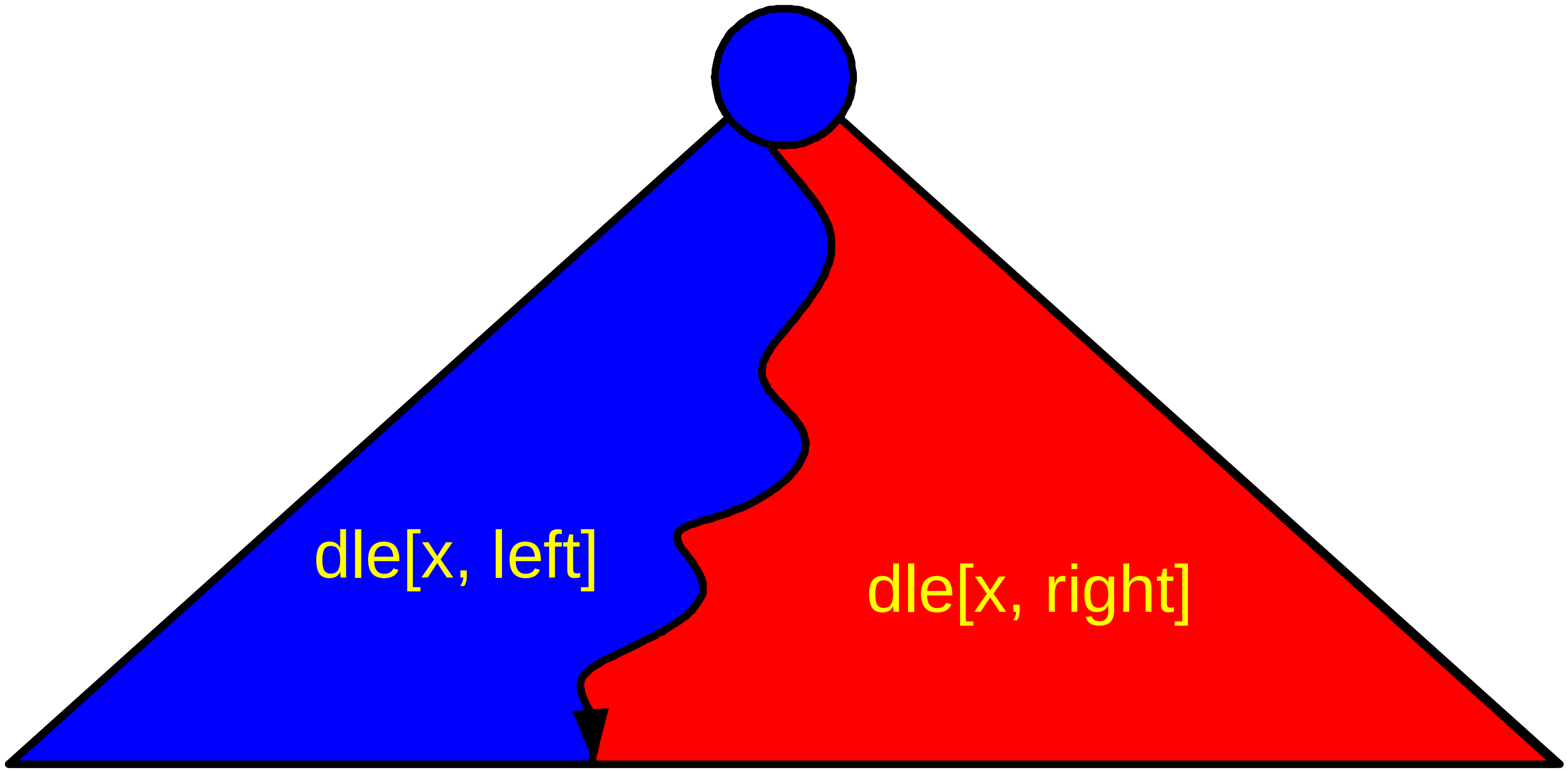
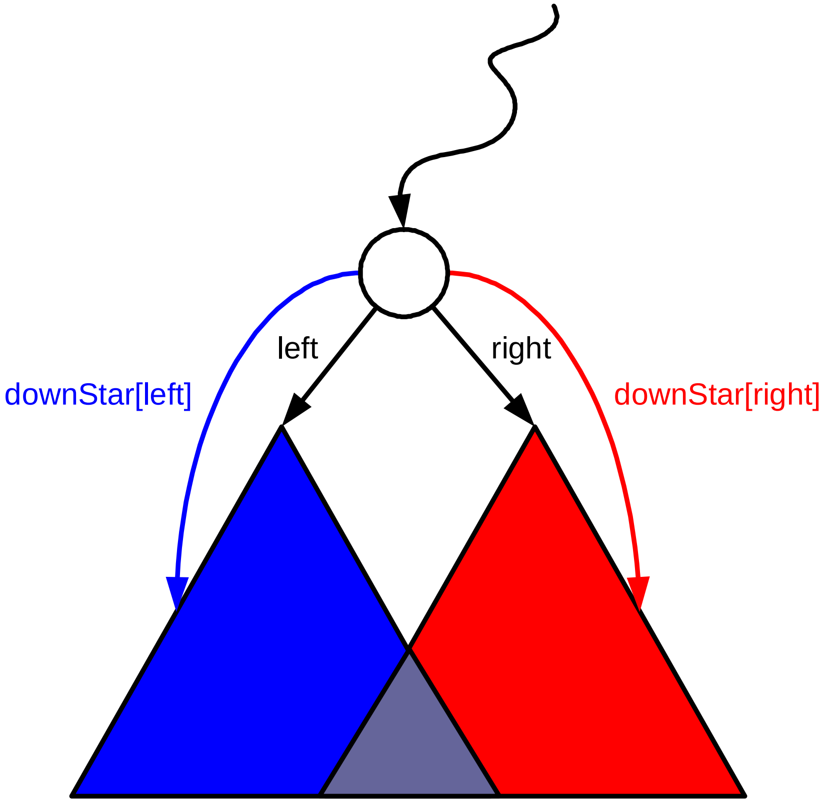
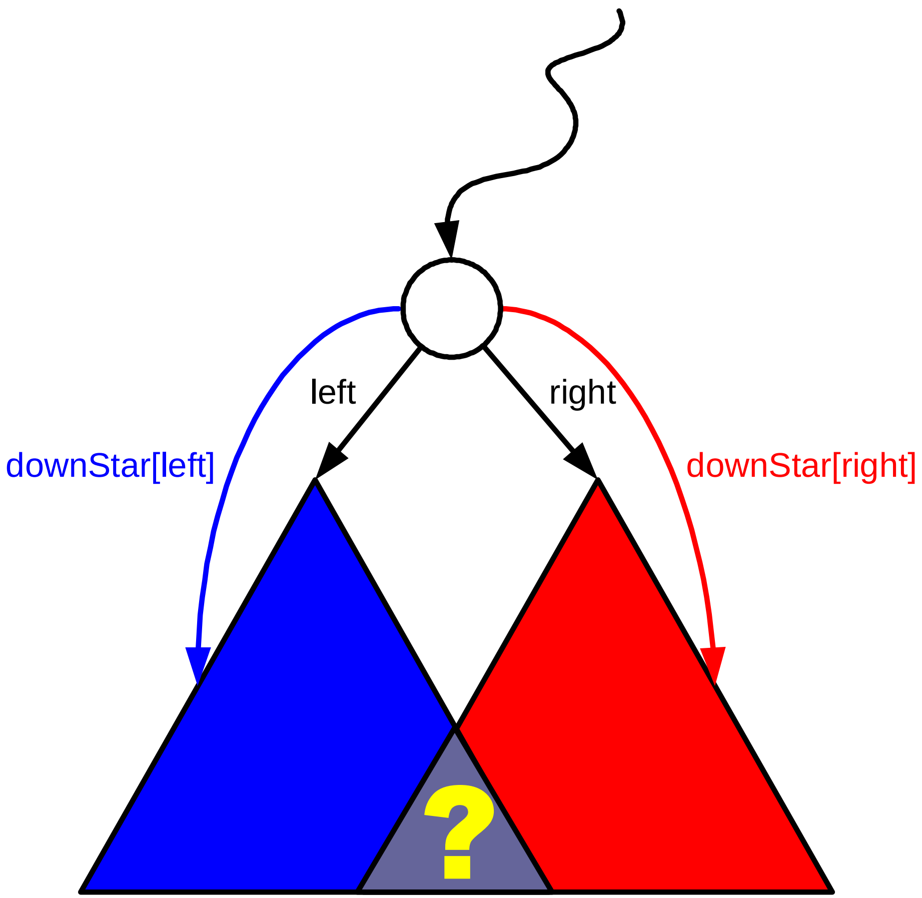


Predicates - downStar(left) / downStar(right)

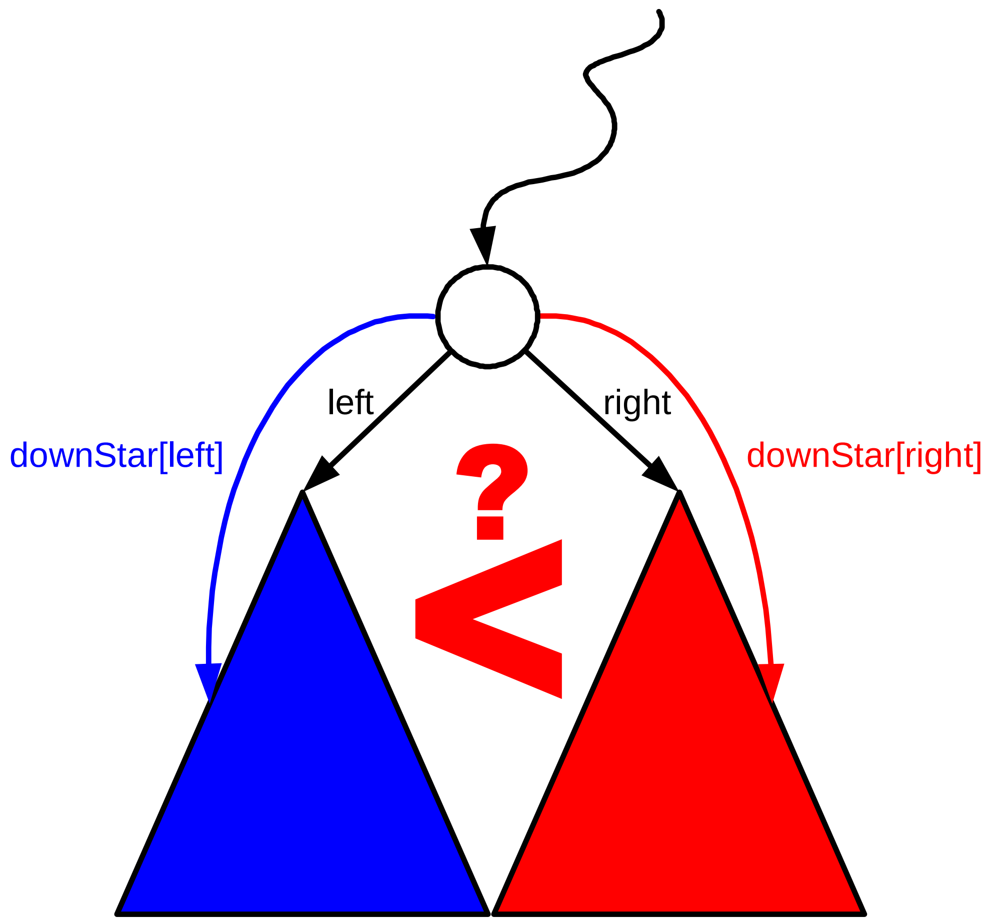
$$down(v_1, v_2) = left(v_1, v_2) \vee right(v_1, v_2)$$
$$downStar[left](v_1, v_2) = \exists v.left(v_1, v) \wedge down^*(v, v_2)$$
$$downStar[right](v_1, v_2) = \exists v.right(v_1, v) \wedge down^*(v, v_2)$$



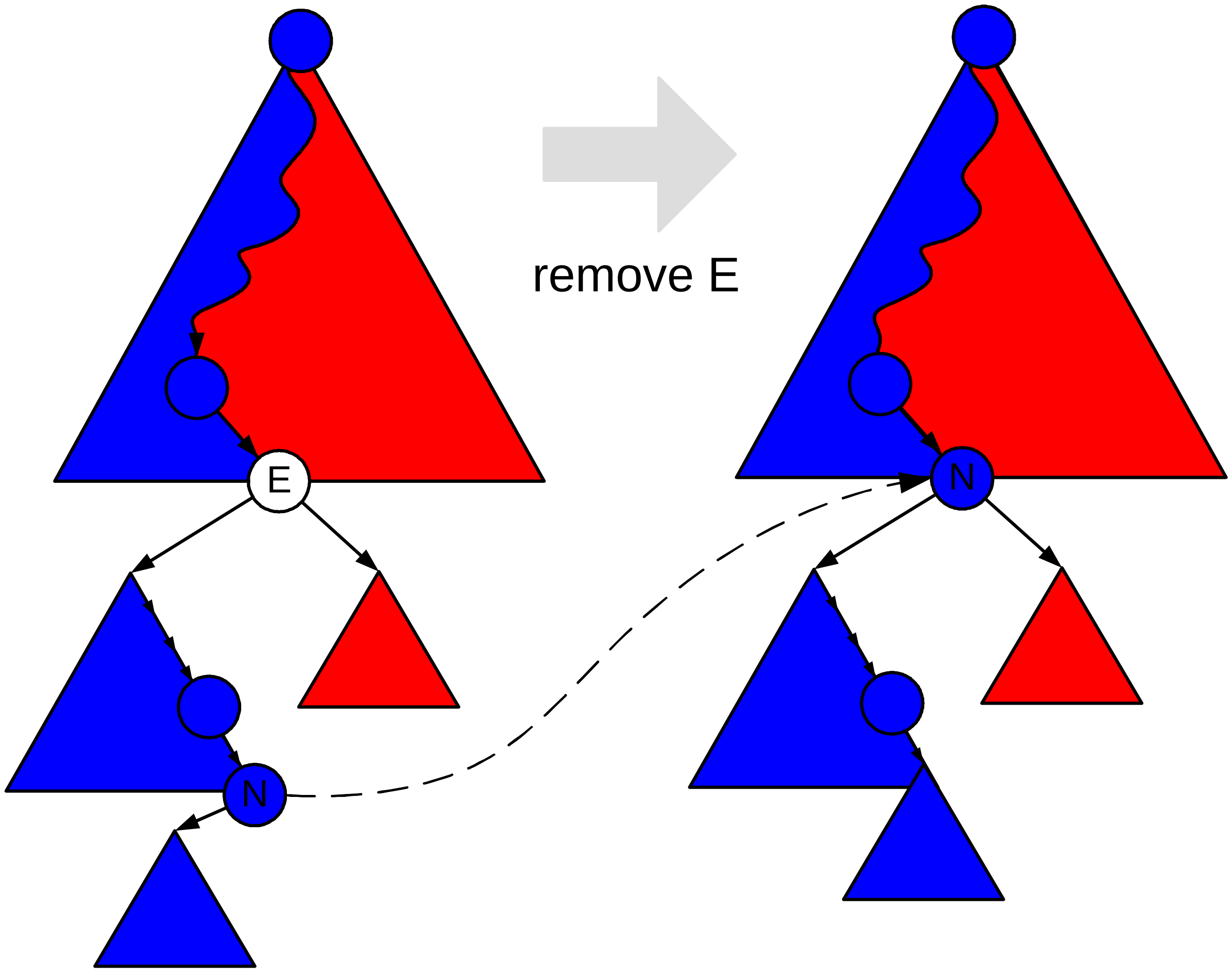
Data Structure Invariants - Is it a tree?



Data Structure Invariants - Is it ordered?



Combined Effect of Predicates



Summary

- Successfully analyzed complex heap-manipulating routines!
- From Axiom to Analysis:
  - Coupled different analyses by instrumentation predicates ($isElement(a, s) \leftrightarrow a \in s$)
  - Remembered old state of predicate to compare it with new state ($isElementOld$)
- Tailoring the abstraction specifically to the data structure was the key: Keeping important ordering and reachability information precise
  $\longrightarrow$ one abstraction for all methods, no loop invariants

The Abstract Data Type Set ✓
A Tree-based Set Implementation ✓
Shape Analysis of the Implementation ✓

Thanks for your attention!

References

(EM85) Hartmut Ehrig and Bernd Mahr. *Fundamentals of Algebraic Specification I*. Springer-Verlag New York, Inc., Secaucus, NJ, USA, 1985.

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(SRW02) Mooly Sagiv, Thomas Reps, and Reinhard Wilhelm. Parametric shape analysis via 3–valued logic. *ACM Trans. Program. Lang. Syst.*, 24(3):217–298, 2002.